Time-Induced Correlations

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Abstract

With this paper, we aim to establish that one can infact induce time-based correations. However, we do not claim that such time-induced correlations can violate CHSH or equivalent inequality.

1 Introduction

There has been a lot of activity in computer modelled simulations of EPR-B experiments, due to a number of recent papers from Accadi, Gill and all. Dr. Gill, had setup a 5000 euro bet against Accardi should he succeed in creating a local model of a simulation, that violates the bell inequality.
Here's a bit of the details. There are 5 computers O, A , B , and X , Y.

- Computer O, is the generator of the 'EPR' correlated source.
- Computer X,and Y, are detectors, that produce a measurement. Each detector can be in one of 2 orientations, let's say, orientation 1 or
- The detector angles are input by computer A and B.
- Dr. Gill, controls computers A and B. He agrees to send equiprobable request for direction combinations of filter orientations
- Computer X and Y, should not communicate about each other's detector orientations, on each trail.
- Computer X and Y, output a binary output: 0 or 1 or -1 or 1, ie, either the particle was detected in spin up state or spin-down state. They cannot output any other value.

After about 15000, he will test to see if they will violate CHSH inequality, which is another partical form of Bell inequality.

2 One way to Correlation

The following method WILL NOT violate CHSH inequality, but it will produce correlation, similar to the quantum mechanical prediction. QM predicts that the correlation between the 2 detectors will be $-\cos\theta_{detector}$.
Now, athough we dont have choice over detector orientations, we can still induce correlations using statistical trick, which i call 'othorgonal attack.'
The idea follows from supposing the existance of othornormal real discrete-time signals, $p(n)$ and $q(n)$, which have the following properties

$$\langle p(n)q(n)\rangle = 0 \tag{1}$$
$$\langle p^2(n)\rangle = 1 \tag{2}$$
$$\langle q^2(n)\rangle = 1 \tag{3}$$

A simple case of p(n) and q(n), that have the above properites, are haar wavelets [-1 -1 1 1] , [1 -1 1 -1]. One can form waveforms really long waveforms p(n), q(n) by repeating [-1 -1 1 1] , [1 -1 1 -1], as in

$$\begin{array}{rcl} p(n) & = & [\;\; -1 \;\; -1 \;\; 1 \;\; 1 \;\; -1 \;\; -1 \;\; 1 \;\; 1 \;\; -1 \;\; -1 \;\; 1 \;\; 1 \;\; -1 \;\; -1 \;\; 1...] \\ q(n) & = & [\;\; 1 \;\; -1 \;\; 1 \;\; -1 \;\; 1 \;\; -1 \;\; 1 \;\; -1 \;\; 1 \;\; -1 \;\; 1 \;\; -1 \;\; 1 \;\; -1 \;\; 1...] \end{array} \tag{4}$$

Take a signal $m_1(n)$, and $m_2(n)$ such that

$$m_1(n) = a\left[\sin\phi\, p(n) + \cos\phi\, q(n)\right] \tag{5}$$
$$m_2(n) = b\left[\sin\theta\, p(n) + \cos\theta\, q(n)\right] \tag{6}$$

where a, b, $\sin\phi$,$\cos\phi$, $\sin\theta$, $cos\theta$ are constant cofficients

Suppose we takee $\langle m_1(n)m_2(n)\rangle$,

$$\begin{aligned} \langle m_1(n)m_2(n)\rangle &= \langle a\left(\sin\phi p(n) + \cos\phi q(n)\right) b\left(\sin\theta p(n) + \cos\theta q(n)\right)\rangle && (7)\\ &= ab\langle \sin\phi\sin\theta p^2(n) + \cos\phi\sin\theta q(n)p(n) && (8)\\ &\quad + \cos\theta\sin\phi q(n)p(n) + \cos\phi\cos\theta q^2(n)\rangle && (9)\\ &= ab[\sin\phi\sin\theta\langle p^2(n)\rangle + \cos\phi\sin\theta\langle q(n)p(n)\rangle + && (10)\\ &\quad \cos\theta\sin\phi\langle q(n)p(n)\rangle + \cos\phi\cos\theta\langle q^2(n)\rangle] && (11) \end{aligned}$$

substituting values for $\langle p^2(n)\rangle$, $\langle q^2(n)\rangle$, $\langle q(n)p(n)\rangle$, etc, we get,

$$= ab\sin\phi\sin\theta + ab\cos\phi\cos\theta \tag{12}$$
$$= ab\cos(\phi-\theta) \tag{13}$$

3 Generating Measurement

METHOD for generating a measurement on the EPR correlated pair locally at computer X and Y. Now, at computer X. We generate 2 streams of measurements, one for filter orientation 1, another for filter orientation 2.

we generate stream X_1,

$$X_1(n) = \text{rand} < a\left(\sin\theta_{x1}p(n) + \cos\theta_{x1}q(n)\right) \tag{14}$$

we generate stream X_2,

$$X_2(n) = \text{rand} < a\left(\sin\theta_{x2}p(n) + \cos\theta_{x2}q(n)\right) \tag{15}$$

rand produces a random number between -1 and 1. $a < b$ return 1, if $a \leq b$, and returns -1 otherwise where n is the trail number.
$X_1(n)$ and $X_2(n)$ are polar binary streams, as per Dr. Gill's request. But to make the othorgonal attack work, the absolute value of the cofficients of $p(n)$ and $q(n)$ must add to less than 1. However, $|\sin\theta| + |\cos\theta| \geq 1$. If one scales it down by adjusting a, to make it work , one no longer will be able to violate bell-inequality.
Now, at computer, Y, we generate, we generate stream Y_1,

$$Y_1(n) = -\left[\text{rand} < b\left(\sin\theta_{x1}p(n) + \cos\theta_{x1}q(n)\right)\right] \tag{16}$$

we generate stream Y_2,

$$Y_2(n) = -\left[\text{rand} < b\left(\sin\theta_{x2}p(n) + \cos\theta_{x2}q(n)\right)\right] \tag{17}$$

When Dr. Gill, sends requests to computer X or Y, to produce, a measurement for his choice of filter orientation, we return local 'measurement' made by the filter.

4 Bell's Inequality: Another form of Shannon's Coding Theorem?

4.1 Draft

Take the Accardi challenge by Dr. Gill. The problem equivalently, is whether detector X can guess and compenstate for the setting of detector Y without actually knowing actual setting at detector Y. This, in general is impossible according to Shannon-Coding Theorem.

4.2 Important Things to Notice

Each detector orientation is choosen independently at X and Y.
Choosing orientation 1 at X is mutually exclusively to orientation 2. So, we can use a single bit or less to encode the state of the oritentation at detector X. Let's call the entropy here H_{gx}.
Choosing orientation 1 at Y is mutually exclusively to orientation 2. So, we can use a single bit or less to encode the state of the orientation at detector Y. Let's call the entropy here H_{gy}.
Dr. Gill has $H_{gx} + H_{gy}$ amount of information he can control to his desire.

Let's call the bit stream generated at orientation 1 at X X_1, orientation 2 at X X_2, orientation 1 at Y Y_1, orientation 2 at Y Y_2. Now, there are four ways this information can paired, at time step n. $X_1(n)$ can be paired against $Y_1(n)$ or $Y_2(n)$, $X_2(n)$ cae be paired against $Y_1(n)$ or $Y_2(n)$ and viseversa.
The detectors do have some prior knowledge. For example, for one, if he writes into X_1, he definitely is not writing into X_2. Similarly, if he is writing into Y_1, he definitely is not writing into Y_2. However, they do not know the correct bit stream he is writing this information into.
Suppose detector X generates a measurement based on Dr. Gill's orientation , it still has no way of controlling Dr. Gill's orientation. More importantly, the detector has no way of controlling which bit stream X_1 and X_2, he is writing this result into. The same could be told of detector Y and the bit streams Y_1 and Y_2 he generates at Y.
You have to make some key notices of the above statements. By controlling which bit stream detector output is written into, he can induce H_{gx} at detector X and H_{gy} at detector Y. The detectors have no say in this matter.

Take the following senarios.
Dr. Gill provides detector X with orientation 1. He generates a bit. And now, detector X is not aware of anything at detector Y. Now, detector X does not know if the bit stream generated at X_1, is going to be paired against either Y_1 or Y_2. Detector X does not have any information channel that is going to tell if Y_1 or Y_2 was used for the orientation. Shannon's Coding Theory tells us that information content of a channel cannot exceed the limit of the channel. Thus, there is always an average H_{gy}-bit uncertainty, at X. We can similarly enumerate through all the other posibilities of detector orientations.

Even if the detectors were aware of the previous states of detector orientations, they will never be able to play catch up, because Dr. Gill is the generating the filter orientations and inducing information content H_{gx} and H_{gy} into the system, at every instant. That would mean that suppose the detectors decided to make up, for the uncertainty in their previous runs, they still miss the uncertainty of information, he introduces in the new iteration. Even if the detectors were able to make up for the uncertainty of information, from the knowledge of the previous iterations, they still will always be one iteration behind, at worst. And thus, will never be able to make up that H_{gx}-bit or H_{gy}-bit.