Non-Lattice Based Universal Computational Models of Physics

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Abstract

The goal of this paper is to introduce non-lattice based computational of physics.The model presented here,is no means, an effort at TOE. The attempt is be true to the following goals:

build a model that doesnt make reference to shape of point.

build a model that doesnt make reference to the number of spatial dimensions.

build a model that is computational universal: that is,one you can program, and run programs with etc.

build a model that has basic properties, required by physics like parity, translation symmtery, rotational symmtery, etc.

build a model that is non-local.

build a model that is compliant with relativity.

Although, the author has not completely succeeded in showing that the model meets all these goals throughly, he does come close.

Attributions

Most of the following ideas, are borrowed from contemporary physics, especially from MTW, and from nascent 'digital' principles and philosophy, from works of Fredkin, Toffeli, Margolus, Cilinger and Klien, and several others.

Thanks

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Contents

1 The Motivation

1.1 Non-coordinate Finite State Autonoma?

Interesting term, but does it mean? Well,non-coordinate refers to that fact that particles, within the simulation, do not explicitly carry a 4-d coordinate point (x,y,z,t), that describe thier location in space-time.
You can even say, that the simulation is coordinate free and that it doesnt refer to any particular coordinate system or basis.
Traditional CA, like GOL or Ed Fredkin's billard ball models, Norm Margolus's lattice gas models, etc, could also be described as coordinate free system, since particles, within the system do not specifically carry coordinates.
But, let's stop here. The shape of a pixel (in the universe) ? What is it? a cube, a square, a hexagon, a circle, a sphere,etc. But wait, why should it, even have a shape?
Can a pixel be just discrete and yet have no shape? Personally, this is where the bet is: although the universe is a discrete system, the pixel, in his opinion, should have no shape. His opinion is not orginal in any sense. In fact, the physics community has been reluctant to accept Ed Fredkin's ideas, for this exact reason.

1.2 One of Several Possible Answers

Now, the question, remains, if one can concieve such a system, and simulate it. Although, the relativistic aspect of that system is sketchy, and the system does not manipulate bits, in a traditional CA sense, the system can be shown to be atleast Newtonian, while at the same time, being true to its orginal goal, by avoiding reference to shape of a point and still be coordinate free.

2 The Setup

The system is setup with N particles. particle i is at s_{ij} invariant distance away from particle j. We have an N^2 matrix elements, most of them carrying redundant information, of each others location. The system is also assumed to be symmetric $s_{ij} = s_{ji}$ and that s_{ii} is undefined and importantly $\widehat{s}_{ij} = -\widehat{s}_{ji}$ .This is the all the information, the system has. It doesnt make any other assumption, as to system, being a 2d, 3d , 4d , 5d or k-d. Just particles, away from each other at some invariant distance.
The author uses a physicist's definition of the term 'invariant distance', meaning that different observers(point-like particles, in our case) will all agree on their measurement of invariant distance, independent of their coordinate system.
Now, the question is how can one simulate the system? So, one starts looking around, as to what should be fundamental, to the system. So, why not start at Newton's first law?

2.1 Newton's First Law

Newton's first law states that every particle continues its state of rest, or state of motion.
If Newton's law is so fundamental to the universe, one starts to question, what it means in the context of computer generated world. After pondering, and taking many wrong paths, the author justifies its importance in the following manner. For Newton's law to be the most natural to the universe, it should be the easiest to implement in the computer simulation.

2.2 An Interesting Observation

Now, let's take a moment of digression, to fundamentals of geometry: the law of cosines. Suppose, we are given 3 points ABC,

```
.....................A...................
..................../.\..................
................./.....\.................
.............../........\................
............./...........\...............
............B-------------C..............
```

According to the law of cosines,

$$AC^2 = AB^2 + BC^2 - 2AB \cdot BC \tag{1}$$

$$AB \cdot BC = \frac{1}{2}(AB^2 + BC^2 - AC^2) \tag{2}$$

The law of cosines, is important for the above reason. One can calculate a DOT product, by only knowing of the distances between the points that make the triangle. And even more importantly, the law of cosines doesnt make any reference to any coordinate system or basis.

3 Method of Simulation

We stop, and we go back to Newton's law. There are N particles.They are away from each other at an coordinate invariant distance s_{ij}. Suppose, we have a particle i. Suppose particle j, changes it distance s_{ij} away from it. Now, it changes its distance away from the other particles, in system. Now, to simplify calculations, suppose, each relative distance is along an axis $\widehat{s}_{ij}$. Let's start with 2-d Newtonian simulation, which is easily extendabke to 3d or even 4d.

We start in absolute Euclidian coordinates, where particle i is at location $(x_i,\, y_i)$. It's distance to particle j is given by

$$s_{ij} = |(x_i - x_j, y_i - y_j)| \tag{3}$$

And suppose, the particle changes by $\Delta\overrightarrow{d} = (\Delta x,\, \Delta y)$. The change in distance from i to particle j is

$$|(x_i - x_j + \Delta x, y_i - y_j + \Delta y)| \tag{4}$$

If Δx and Δy are small, then

$$\begin{aligned}
& |(x_i - x_j + \Delta x, y_i - y_j + \Delta y)|^2 & (5)\\
= \; & (x_i - x_j)^2 + 2(x_i - x_j)\Delta x + \Delta x^2 & (6)\\
+ \; & (y_i - y_j)^2 + 2(y_i - y_j)\Delta y + \Delta y^2 & (7)\\
= \; & (x_i - x_j)^2 + (y_i - y_j)^2 & (8)\\
+ \; & 2(x_i - x_j)\Delta x + 2(y_i - y_j)\Delta y & (9)\\
= \; & s_{ij}^2 + 2\overrightarrow{s_{ij}} \cdot \Delta \overrightarrow{d} & (10)
\end{aligned}$$

Suppose we take the above expression and do a bit of rewriting.

$$\begin{aligned}
= \; & s_{ij}^2 + 2\overrightarrow{s}_{ij} \cdot \Delta \overrightarrow{d} & (11)\\
= \; & s_{ij}^2 (1 + 2\tfrac{\overrightarrow{s}_{ij} \cdot \Delta \overrightarrow{d}}{s_{ij}^2}) & (12)\\
= \; & s_{ij}^2 (1 + 2\tfrac{\widehat{s}_{ij} \cdot \Delta \overrightarrow{d}}{s_{ij}}) & (13)\\
& & (14)
\end{aligned}$$

taking $\surd$ and expanding by first order approximation gets,

$$\begin{aligned}
= \; & s_{ij} \sqrt{1 + 2\tfrac{\widehat{s}_{ij} \cdot \Delta \overrightarrow{d}}{s_{ij}}} & (15)\\
= \; & s_{ij} (1 + \tfrac{\widehat{s}_{ij} \cdot \Delta \overrightarrow{d}}{s_{ij}}) & (16)\\
= \; & s_{ij} + \widehat{s}_{ij} \cdot \Delta \overrightarrow{d} & (17)
\end{aligned}$$

In the general case, one would obtain

$$\frac{ds_{kj}}{d\tau} = \widehat{s}_{kj} \cdot \overrightarrow{u_j} \tag{18}$$

where u_j is the velocity of the particle j. Now, within our system, one cannot define u like a vector, for example, as in (u_t,u_x,u_y, u_z). Instead, one can write a **algebraic** rule for u, in terms of direction of other particles,for example, like in

$$\overrightarrow{u} = a\widehat{s}_{pq} + b\widehat{s}_{lm} + c\widehat{s}_{na} + ... \tag{19}$$

where a,b,c etc are constants.
NOTE: We will interpret the above rule to mean that the observer can only make measurements, in terms of other particles in systems.

3.1 Implementation

The rule by itself, is ambigious. For example, given a bunch of particles, j, k, etc, the order, in which, one applies the rules, is not complelely clear. Take

$$\frac{ds_{kj}}{d\tau} = \widehat{s}_{kj} \cdot \overrightarrow{u_j} \tag{20}$$

$$\frac{ds_{jk}}{d\tau} = \widehat{s}_{jk} \cdot \overrightarrow{u_k} \tag{21}$$

where each rule has to change s_{jk} and s_{kj}, just to be consistant.

$$\frac{ds_{kj}}{d\tau} = \widehat{s}_{kj} \cdot \overrightarrow{u_j} \tag{22}$$

$$\frac{ds_{kj}}{d\tau} = -\widehat{s}_{kj} \cdot \overrightarrow{u_k} \tag{23}$$

But which rule goes first? or do they both, go first? One non-ambigious way to sort this problem, is by assuming that the s_{jk} on the RHS, always comes from the previous step. So, the actual implementation, should behave more like,

$$\frac{ds_{kj}}{d\tau} = \widehat{s}_{kj} \cdot [\overrightarrow{u_j} - \overrightarrow{u_k}] \tag{24}$$

3.2 An Obvious Coordinate Problem?

Now, if you have already noticed,$\widehat{s}_{pq}$ is a vector,and it requires direction components in terms of a coordinate basis. So, you would be inclined to ask, how we can get away with not knowing the coordinates, if our rules make reference to them. Notice that, in

$$\frac{ds_{kj}}{d\tau} = \widehat{s}_{kj} \cdot \overrightarrow{u_j} \tag{25}$$

s_{kj} is a scalar, and $\widehat{s}_{kj} \cdot \overrightarrow{u_j}$ is also a scalar. And this where the law of cosine comes to our help. It allows us to calculate the dot product $\widehat{s}_{kj} \cdot \overrightarrow{u_j}$ without actually knowing about the components of s_{kj}, or $\overrightarrow{u_j}$, but by only using the invariant distances , with which, we are simulating.

Now, suppose particle i moves away from j, at velocity $\overrightarrow{u_j}$. And suppose, $\overrightarrow{u_j}$ is given by

$$\overrightarrow{u_j} = a\widehat{s}_{iq} + b\widehat{s}_{ip} + c\widehat{s}_{ir} \tag{26}$$

where q, p and r are particle indecies. Now, we know the rate of distance s_{kj} increase, is given by

$$\begin{aligned} \widehat{s}_{kj} \cdot \overrightarrow{u_j} &= \overrightarrow{u_j} \cdot \widehat{s}_{kj} && (27) \\ &= a\widehat{s}_{iq} \cdot \widehat{s}_{kj} + b\widehat{s}_{ip} \cdot \widehat{s}_{kj} + c\widehat{s}_{ir} \cdot \widehat{s}_{kj} && (28) \end{aligned}$$

Now, to calculate $\widehat{s}_{iq} \cdot \widehat{s}_{kj}$, we first take,

$$\widehat{s}_{iq} = \frac{\overrightarrow{s_{kq}} - \overrightarrow{s_{ki}}}{s_{iq}} \tag{29}$$

Now to calculate $\overrightarrow{s_{kq}} \cdot \widehat{s}_{kj}$, we can just reuse our law of cosine relation.

$$\begin{aligned} \overrightarrow{s_{kq}} \cdot \widehat{s}_{kj} &= \frac{\overrightarrow{s_{kq}} \cdot \overrightarrow{s_{kj}}}{s_{kj}} && (30) \\ &= \frac{1}{2s_{kj}}(s_{kq}^2 + s_{kj}^2 - s_{qj}^2) && (31) \\ & && (32) \end{aligned}$$

Now, to calculate $\widehat{s}_{iq} \cdot \widehat{s}_{kj}$, we can just

$$= \frac{1}{2s_{kj}s_{iq}}(s_{kq}^2 + s_{kj}^2 - s_{qj}^2 - s_{ki}^2 - s_{kj}^2 + s_{ij}^2) \quad (33)$$
$$= \frac{1}{2s_{kj}s_{iq}}(s_{kq}^2 - s_{qj}^2 - s_{ki}^2 + s_{ij}^2) \quad (34)$$
$$= \frac{1}{2s_{kj}s_{iq}}(s_{kq}^2 - s_{ki}^2 + s_{ij}^2 - s_{qj}^2) \quad (35)$$

So, when we work everything out, we have a system, that allows us to simulate particles, (atleast for the Newtonian case), without expliciting describing in the system in a particular coordinate basis.

4 Properties

4.1 Non-Local!

Interestingly yes. The system is non-local, because the locality in the system, and the locality in the world generated by the system are different things, and so, it definitely does accommodate for Bell's theorem. Not only is that true, the model inherently has non-local elements, incorporated into it and so, will give specific predictions, as of their effects.

4.2 Non-Linear!

Given the rules where

$$\frac{ds_{kj}}{d\tau} = \widehat{s}_{kj} \cdot \overrightarrow{u_j} \quad (36)$$

where

$$\overrightarrow{u_j} = a\widehat{s}_{ab} + b\widehat{s}_{cd} + c\widehat{s}_{ef} + ... \quad (37)$$

Suppose $\overrightarrow{u_j} = \widehat{s}_{iq}$. Immediately, we will have

$$\frac{ds_{kj}}{d\tau} = \frac{1}{2s_{kj}s_{iq}}(s_{kq}^2 - s_{ki}^2 + s_{ij}^2 - s_{qj}^2) \quad (38)$$

such a system is non-linear. Now, suppose s_{iq} depends on s_{kj}, such a system is non-linear, and even worse.

4.3 Not a Traditional CA!

There are some fundemental issues, that needs to be addressed. Take, the law of cosines.

$$AB\ BC\cos(\angle ABC) = \frac{1}{2}(AB^2 + BC^2 - AC^2) \quad (39)$$

Calculation of AB^2 and divisin by $AB\ BC$ is pretty inexpensive operation, compared to taking the $\cos(\angle ABC)$. But the use of a mathematical formula, for the simulation, goes against the whole philosophy of CA systems. There is a famous debate between Johz Baez and Ed Fredkin, as to whether CAs are 'mathematical'. There is something even more fundamental than these mathematical rule, and the mathematics must be emergent.

5 Universality and Momentum Conservation

The system could be made computational universal, by adding laws of momentum conservations and adding two types of particle and their interactions. The author does not claim that these particle directly correspond to particles, in physics
One of Ed Fredkin and Toffeli's best papers is 'Conservative Logic'. The paper can be found at

`http://www.digitalphilosophy.org/download_documents/ConservativeLogic.pdf`

There, they dicuss, how one can build a reversible and universal Fredkin gate, from just billard ball interactions. Although, we will depend their approach, to show that the system, is universal, we will not depend on their arguement to show that it is time-reversible.
In Fredkin and Toffeli's paper, if a ball is at location (t,x,y,z), there is a logic 1 in that location. If there no ball is at location (t,x,y,z), it corresponds to logic 0. Fredkin and Toffeli realize their reversible universal gate, by having the balls collide at 90 degrees at each other, and using the law of conservation of momentum, between the balls and between the balls and objects called mirrors. In the following sections,we will outline a method, for detecting collisions and conserving momentum

5.1 Detecting An Interaction

To detect collisions between particles, we check their s_{ij}. If it is within an ϵ, they have collided. Relativistically, this solution is bound to fail,since relativistic $s_{ij} = 0$ would refer to a null path. Instead, the best way to detect a collision is to check entries of s_{ix} with s_{jx} and s_{yi} with s_{yj}. If they are all equal, to about some ϵ, they are particles, occuping the same space. So, we have a collision. If more than two particles, mange to occupy the same space, precedence could be given based on their column index position, in the matrix, but we will not make. Such a rule would be non-ambigious. But we will, for now, not make any such unwanted assumptions about the model. Ambiguities would be automatically dealt, by adding many-world rules to the mixture.

5.2 Conserving Momentum

Suppose, if we assume that all particles are composed of unit mass, and suppose, if we swap $\overrightarrow{u_i}$ with $\overrightarrow{u_j}$ momentum would be conserved.
We assume that in addition to s_{ij}, particles also carry velocity information, in a seperate matrix.
And the interaction would be reversible. Now, to incorporate the concept of a mirror, we have introduce, 2 kinds of particles, call it, soft and hard. When a soft particle, collides with a soft particle, we swap the $\overrightarrow{u_i}$ and $\overrightarrow{u_j}$. When a hard particle, collides with a hard particle, we swap the $\overrightarrow{u_i}$ and $\overrightarrow{u_j}$. Such rules also have the property that the angle before the collision and that after the collisions are the same. So, if the balls initially collided at

90 degrees to each other, they will bounce back at 90 degrees,as required by Ed Fredkin and Tofelli's model.

5.3 An ϵ Does Make a Difference

When 2 balls collide, what difference is there, between the ball going about their paths and exchanging momentum during collision? For one simple difference, the ϵ. There is one other complicated artifact of exchanging momentum during collision, that would not matter even if ϵ is zero or very large. We will get back to it, later.
Let's start with Fredkin and Toffeli's realization of their interaction gate.

```
.............................................................
.............................................................
...............p.(*).........(*).pq.........................
...................\.........'...............................
....................\.......'(*).p'q.........................
.....................\.....'/................................
......................\...'/.................................
.......................(*)/..................................
.......................(*)\..................................
....................../...'\.................................
...................../.....'\................................
..................../.......'(*).q'p.........................
.................../.........'...............................
...............q.(*)..........(*).pq.........................
.............................................................
.............................................................
```

This diagram, is a testament to Fredkin and Toffeli's genius, for one specific reason: The difference between $p'q$ and pq and between $q'p$ and pq, is their location in space!! and that is made possible, because of the lattice structure, it is built on. Athough, our setup, does not have a lattice structure, it does make room for a very small, but finite ϵ. As long as ϵ doesnt vanish, a balls exchanging momentum during collision, will have spatial trajectories different than balls passing through each other.

5.4 Mirror Mirror On the Ball

When a soft particle, collides with a hard particle, or vise versa, we reflect the soft particle, away from the hard particle, in the following manner.
Suppose $\overrightarrow{u_l}$ be velocity of the soft particle and $\overrightarrow{u_h}$ be velocity of the hard particle. Then,

$$\overrightarrow{u_{l\,new}} = (\overrightarrow{u_l} \cdot \widehat{u}_h)\,\widehat{u}_h - (\overrightarrow{u_l} - (\overrightarrow{u_l} \cdot \widehat{u}_h)\,\widehat{u}_h) \tag{40}$$

$$= 2\,(\overrightarrow{u_l} \cdot \widehat{u}_h)\,\widehat{u_h} - \overrightarrow{u_l} \tag{41}$$

$$= \frac{2}{u_h^2}\,(\overrightarrow{u_l} \cdot \overrightarrow{u_h})\,\overrightarrow{u_h} - \overrightarrow{u_l} \tag{42}$$

And we leave the heavy particle unchanged. It could be possible that the heavy particle, is itself a statistical collection of light particles and the

rule does not have to be specifically implemented.
The above rules are not completely unambigious. For example, take the rule for reflection.

$$\overrightarrow{u}_{l\ new} = 2\frac{1}{u_h^2}\left(\overrightarrow{u_l}\cdot\overrightarrow{u_h}\right)\overrightarrow{u_h} - \overrightarrow{u_l} \tag{43}$$

Now,after the soft ball collides with the hard one, since u_h, in its original form ,is an **algebraic** expression, as opposed to a number. We can either keep track of the resultant simplified algebraic expression of $\frac{2}{u_h^2}\left(\overrightarrow{u_l}\cdot\overrightarrow{u_h}\right)\overrightarrow{u_h}$ or, to keep the expressions neat and tidy, we can just evaluate $\frac{2}{u_h^2}\left(\overrightarrow{u_l}\cdot\overrightarrow{u_h}\right)$ numerically at that instant, and project it onto **algebraic** expression $\overrightarrow{u_h}$. We choose the neat implementation.
But given those rules, Fredkin's billard ball interaction model, becomes a single instance of these rules. Now, using Fredkin and Toffeli's methods,one can build complex logic gates from them.

6 A Relativistic Model

For a system to be relativistic, it must meet 2 simple requirements.
Requirement 1 Dot-product must have Minkowski signature, that is

$$\overrightarrow{da} \cdot \overrightarrow{db} = -da_0 db_0 + da_1 db_1 + da_2 db_2 + da_3 db_3 \tag{44}$$

and
Requirement 2 Space-time seperation remains the same, in any frame of reference,

$$-dt^2 + dx^2 + dy^2 + dz^2 = -dt'^2 + dx'^2 + dy'^2 + dz'^2 \tag{45}$$

where dt, dx, dy, dz are measured in one frame and dt', and dx', dy', and dz' are from other frames.

6.1 Law of Cosines in Flat Minkowski space

Surprisingly, the law of cosines is also valid, in flat-space relativistic 4d-systems.
Pick a frame and take space-time vectors $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$. Suppose $\overrightarrow{a} = (a_0, a_1, a_2, a_3)$ and $\overrightarrow{b} = (b_0, b_1, b_2, b_3)$ and $\overrightarrow{c} = (a_0 - b_0, a_1 - b_1, a_2 - b_2, a_3 - b_3)$. Now,

$$a^2 = -a_0^2 + a_1^2 + a_2^2 + a_3^2 \tag{46}$$

$$b^2 = -b_0^2 + b_1^2 + b_2^2 + b_3^2 \tag{47}$$

and

$$\begin{aligned} c^2 &= (\overrightarrow{a} - \overrightarrow{b})^2 && (48) \\ &= -(a_0 - b_0)^2 + (a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 && (49) \\ &= -a_0^2 - b_0^2 + 2a_0 b_0 + a_1^2 + b_1^2 - 2a_1 b_1 + ... && (50) \\ &= a^2 + b^2 - 2a \cdot b && (51) \end{aligned}$$

And more importantly, suppose, a^2, b^2 and c^2 were precomputed using the Minkowski metric, then $a \cdot b$ computed by taking $\frac{1}{2}\left(a^2 + b^2 - c^2\right)$ has the Minkowski metric signature.

$$\begin{aligned} \frac{1}{2}\left(a^2 + b^2 - c^2\right) &= a \cdot b && (52) \\ &= -a_0 b_0 + a_1 b_1 + a_2 b_2 + a_3 b_3 && (53) \\ & && (54) \end{aligned}$$

6.2 Universal Proper Time? No!

Given our model,

$$\frac{d}{d\tau} s_{ij} = \widehat{s}_{ij} \cdot u_j \tag{55}$$

since the equation involves proper time τ, you may be inclined to think, that we have to pick a particular particle k. And simulate all the other particles, in that system, by rewriting any RHS term $\overrightarrow{s_{qm}}$ into $\overrightarrow{s_{km}} - \overrightarrow{s_{kq}}$. This must be noted that if s_{ij} is frame invariant, it implies that ds_{ij} is frame invariant, and it naturally it follows that $\widehat{s}_{ij} \cdot \overrightarrow{u_j} d\tau$ is frame invariant. Given that our system is discrete, naturally it is possible to replace '$\overrightarrow{u_j}$ rules', with '$\overrightarrow{\Delta_j}$ rules', which has the same unit as distance. Then we can take

$$\overrightarrow{\Delta_j} = a\overrightarrow{s_{iq}} + b\overrightarrow{s_{gh}} + c\overrightarrow{s_{xy}} + \ldots \text{or even} \tag{56}$$

$$\overrightarrow{\Delta_j} = a\widehat{s}_{iq} + b\widehat{s}_{gh} + c\widehat{s}_{xy} + \ldots \tag{57}$$

we can update every particle's space-time seperation, away from every other particle, in its own frame.

6.3 Initial Condition

We take, by assumption that s_{ij} was initially built using the Minkoswki metric. However, this requirement has a possibility of emerging for a discrete system, from random initial conditions.
The proof for the inductive arguement below, works at its best, if the system is simulated by a continous differential equation. However, if the system was simulated with a discrete difference equation, the dimensionalty or even the metric signature of the system, does vary slightly over time.

6.4 Induction from n to n+1

If we assume that s_{ij} at time step n was built using Minkoswki metric. Then take the equation

$$\Delta s_{kj} = \widehat{s}_{kj} \cdot \overrightarrow{\Delta}_j \tag{58}$$

Suppose, $\overrightarrow{\Delta_j} = \widehat{s}_{iq}$, then calculation of

$$\widehat{s}_{kj} \cdot \widehat{s}_{iq} = \frac{1}{2 s_{kj} s_{iq}} (s_{kq}^2 - s_{ki}^2 + s_{ij}^2 - s_{qj}^2) \tag{59}$$

complies with requirement (1), by virtue of s_{kq}, s_{ki}, s_{ij} and s_{qj} having a Minkowski signature , as discussed in the previous section. However, s_{kj}, at iteration n+1, given by an increase by Δs_{kj}, computed from the above, is only approximately Minkoswki compliant.
This is because the term $\widehat{s}_{kj} \cdot \Delta_j$ is an first-order approximation itself, and is only throughly valid as Δ_j approaches 0.
Requirement 2, is naturally statisfied, by this rule, since, we are directly manipulating invariant space-time seperation s_{ij}.

6.5 Visualizing The System in a Particular Observer's Frame

Every observer(point-like particle,in our case) caries a terad(coordinate system) with him(it), as illustrated by the following graph, at time proper instant 0 and 1.

```
...............................................
....................|.y(1).....................
....................|..........................
....................|..........................
....................*------.x(1)...............
................. /'...........................
..........\.y(0)../'z(1).......................
...........\....'..............................
............\..'...............................
............*.------.x(0)......................
...........'./.................................
..........'./..................................
........'../.z(0)..............................
......'........................................
```

Now, although, we are only operating on distances s_{ij}, it would interesting, if we can see, how the other particles are projected onto an another particles coordinate system. Although, intially, this may seem like a complicated process, since we are working with scalars s_{ij}. This is a fairly easy task, if one does the homework.

6.6 Embedding a Terad

We start with in observer O's frame. Take his axis $\widehat{o}_t$, $\widehat{o}_x$, $\widehat{o}_y$, and $\widehat{o}_z$[1]. Our $\widehat{o}_t$, $\widehat{o}_x$, $\widehat{o}_y$, $\widehat{o}_z$ get their own columns and rows in the matrix. We assume that these are physical rods of unit length, justapoxed to him. Now, we calculate every other particles j's space-time seperation, away from these rods and store them in $s_{(ox,j)}$, $s_{(oy,j)}$, $s_{(oz,j)}$, $s_{(ot,j)}$, respectively.
Throughout the simulation, $\widehat{o}_t$, $\widehat{o}_x$, $\widehat{o}_y$, and $\widehat{o}_z$ are maintained stationary to our observer. Now, suppose, we would like to visualize the simulation of the entire system, in his frame, all we have to do, is take the DOT product of other particle's $\overrightarrow{s_{oj}}$ with $\overrightarrow{s_{ot}}$, $\overrightarrow{s_{ox}}$, $\overrightarrow{s_{oy}}$, and $\overrightarrow{s_{oz}}$,by relying on the law of cosines.

6.7 Interial Frame

Atleast for the continious case of the equation, a bunch of particles i and j, are in the same interial frame, if $u_j - u_i$ and successive collisions, of j and i with other particles, leave $u_j - u_i$ leave it algebraically equivalent to the one, before the collision.

[1]There are some problems to be addressed later

6.8 Giving It A Spin

It is a commonly accepted fact that spin angular momentum, emerges when relativity and quantum mechanics are combined. So, if one can show that one can build Quantum Mechanics, using this system, as one of its parts, spin would be an emergent property within that system.

7 Conformance To Basic Symmetries

7.1 Parity

The system and its reflection about any plane, would have the same values of representation, and would be indistingushable from each other,because distances, do not change, during reflection

7.2 Static Angular Symmetry

Since rotations leave the distance unchanged, the values of the system of s_{ij} and a rotated counterpart would also be indistingushable from other other.

7.3 Static Translational Symmetry

Since translations leave the distance unchanged, the values of the system of s_{ij} and a translated counterpart, are indistingushable from other other.

8 Skipping Space, Jumping Time and Rounding off

That's all is required for all dynamic symmetries to be violated. If the system is simulated,using a discretized form of the equation, the system would not have a perfect invariance of motion.

8.1 Approximate Time Reversal

Ideally speaking, to simulate the system,backward in time, all one has to do, is just negate all velocities and simulate the particles. If the equation is simulated, using a perfectly continous system, it would be perfectly reversible. However, a discretized version of the equation, would only approximately make the system go back in reverse, in the same way,it went forward.

8.2 Approximate Translation And Rotation

Given that the system is discrete, any form of translation or rotation, is only going to be approximate.

8.3 Noether's Theorem

Nother's Theorem is statified in the limit, when the discrete system is good enough to look and behave like a continous one.

References

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