Chapter 4: Extended Complex Probability Analysis

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Abstract

The goal of this chapter is to generalize probability to complex numbers. We prove that complex probability analysis developed in the previous chapter and the probability analysis prevalent in Quantum Mechanics can be derived from the axiomatic systems, developed in the chapter.

1 Construction

We begin by assuming that numerical value 0 is **logic 0** and and numerical value 1 is **logic 1**. Now, given a logic variable x, $1-x$ is the inversion of x. To contruct an AND gate, we just take xy. To construct an OR gate, we construct it, from the sum of products $xy + (1-x)y + (1-y)x$. So far, so good and, nothing new.

Axiom 1 *Given independent binary variables x_i, we start by assuming that probability p_{ik}, is either a complex or real number or even for that matter, a matrix or a tensor. p_{i0} is the probability amplitude for $x_i = 0$. p_{i1} is the probability amplitude for $x_i = 1$.*

Axiom 2 $|p_{i0}|^n + |p_{i1}|^n = 1$.

Axiom 3

$$\int_{-\infty}^{\infty} dx \delta(x-a) f(x) = f(a) \tag{1}$$

Axiom 4 *We assume that* **logic 0** *and* **logic 1** *are mutually exclusive events. Another way to write the above condition is that*

$$\int_{-\infty}^{\infty} dx \delta(x)\delta(x-1)f(x) = 0 \tag{2}$$

Axiom 5 *Given that* $p(\mathrm{E})$ *is the amplitude of the event* E, $|p(\mathrm{E})|^n$ *is the probability that event* E *is true.*

Axiom 6 *The generator of probability amplitude, is operator*

$$\prod_i \int_{-\infty}^{\infty} dx_i \left[p_{i1}\delta(1-x_i) + p_{i0}\delta(x_i)\right] \tag{3}$$

When it acts on a set of logical conditions $\mathrm{C}(\mathbf{X})$, it becomes

$$\left[\prod_i \int_{-\infty}^{\infty} dx_i p_{i1}\delta(1-x_i) + p_{i0}\delta(x_i)\right] \mathrm{C}(\mathbf{X}) \tag{4}$$

We will call it the DeMorgan generator, since it avoids double/triple-counting for the same event.
Take $\mathrm{C}(\mathbf{X}) = x^n$,

$$\int_{-\infty}^{\infty} dx \left[p_{x0}\delta(x) + p_{x1}\delta(1-x)\right] x^n = p_{x1} \tag{5}$$

1.1 OR logical condition

Suppose take the OR logical condition, $\mathrm{C}(\mathbf{X}) = x_0x_1 + (1-x_0)x_1 + x_0(1-x_1)$.

$$\left[\prod_i \int_{-\infty}^{\infty} dx_i p_{i1}\delta(1-x_i) + p_{i0}\delta(x_i)\right] \left[x_0x_1 + (1-x_0)x_1 + x_0(1-x_1)\right] \tag{6}$$

it becomes

$$p_{01}p_{11} + p_{00}p_{11} + p_{01}p_{10} \tag{7}$$

The probability for the logical OR condition, is

$$|p_{01}p_{11} + p_{00}p_{11} + p_{01}p_{10}|^n \tag{8}$$

Similary, for the inverse of the OR logical condition, $1 - \mathrm{C}(\mathbf{X})$, the probability amplitude is

$$p_{00}p_{10} \tag{9}$$

And the probability is

$$|p_{00}p_{10}|^n \tag{10}$$

2 Standard Probability Theory

2.1 Parker-Mcklusky Equations

From the rules derived above, one can see that Parker-Mcklusky rules and equations, is special case where $n = 1$ and p is real.

2.2 Quantum Mechanical Probability Amplitudes

From the rules dervied above, one can see that properties of Quantum Mechanical amplitudes, is a special case of $n = 2$ and a complex p.

3 Suggestion of the $\sqrt[n]{NOT}$ gate

Using Quantum Mechanical probability amplitudes, Dr. David Detusch was able to construct $\sqrt{NOT}$ gate. And by cascading 2 such gates, he was able to get the NOT gate. The formulation above, suggests the existance of $\sqrt[n]{NOT}$ gate.