Fermat's Last Theorem: In Defense of Fermat

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Abstract

To point of this paper, is to look at Fermat's defense, if he really did have a proof, what it might involve and look like. At his disposable were Calculus, Binomial theorem and rudimentary number theory.

1 Fermat's Last Theorem

According to Fermat's Last Theorem, equation of the form

$$X^n + Y^n = Z^n \tag{1}$$

cannot have distinct integer solutions for X,Y and Z, for $n > 2$.

Proof. Differentiating the above expression by some variable t, we get

$$nX^{n-1}\frac{dX}{dt} + nY^{n-1}\frac{dY}{dt} = nZ^{n-1}\frac{dZ}{dt} \tag{2}$$

One has to notice that the above expression would not make a difference, if we replace $n\frac{dX}{dt}$, $n\frac{dY}{dt}$ and $n\frac{dZ}{dt}$ by labels a', b', c'.

$$a'X^{n-1} + b'Y^{n-1} = c'Z^{n-1} \tag{3}$$

Normalizing the equation by c', would yeild,

$$aX^{n-1} + bY^{n-1} = Z^{n-1} \tag{4}$$

Suppose, we differentiate the system,

$$a'X^{n-1} + b'Y^{n-1} = c'Z^{n-1} \tag{5}$$

again by another variable t'.

$$\frac{da'}{dt'}X^{n-1} + a'(n-1)X^{n-2}\frac{dX}{dt'} + \frac{db'}{dt'}Y^{n-1} + b'(n-1)Y^{n-2}\frac{dY}{dt'} = \frac{dc'}{dt'}Z^{n-1} + c'(n-1)Z^{n-2}\frac{dZ}{dt'} \tag{6}$$

Replacing $\frac{da'}{dt'}$, $(n-1)\frac{dX}{dt'}$, and others with unknowns d , e, f, g, m and n, we would end up with

$$dX^{n-1} + eX^{n-2} + fY^{n-1} + gY^{n-2} = mZ^{n-1} + nZ^{n-2} \tag{7}$$

Set $\omega_x = \frac{X}{Z}$ and $\omega_y = \frac{Y}{Z}$. Now, FLT becomes

$$\omega_x^n + \omega_y^n = 1 \tag{8}$$
$$a\omega_x^{n-1} + b\omega_y^{n-1} = 1 \tag{9}$$

Rewriting the above statements, we end up with,

$$\omega_x^n = 1 - \omega_y^n \tag{10}$$
$$a\omega_x^{n-1} = 1 - b\omega_y^{n-1} \tag{11}$$

Divinding one expression by another we get,

$$\omega_x/a = \frac{1-\omega_y^n}{1-b\omega_y^{n-1}} \tag{12}$$

Rewriting the above expression, we have

$$\omega_x = a\frac{1-\omega_y^n}{1-b\omega_y^{n-1}} \tag{13}$$

Substituing the above expression into $\omega_x^n + \omega_y^n = 1$.

$$a^n\frac{(1-\omega_y^n)^n}{(1-b\omega_y^{n-1})^n} = 1-\omega_y^n \tag{14}$$
$$a^n(1-\omega_y^n)^{n-1} = (1-b\omega_y^{n-1})^n \tag{15}$$

If one can solve the above polynomial equation, one can obtain the expression for ω_y. Similarly, we can obtain a polynomial equation e for ω_x,

$$b^n(1-\omega_x^n)^{n-1} = (1-a\omega_x^{n-1})^n \tag{16}$$

Now, subtituting $X = \omega_x Z$, and $Y = \omega_y Z$,

$$d\omega_x^{n-1}Z^{n-1} + e\omega_x^{n-2}Z^{n-2} + f\omega_y^{n-1}Z^{n-1} + g\omega_y^{n-2}Z^{n-2} = mZ^{n-1} + nZ^{n-2} \tag{17}$$
$$d\omega_x^{n-1}Z + e\omega_x^{n-2} + f\omega_y^{n-1}Z + g\omega_y^{n-2} = mZ + n \tag{18}$$
$$\left[e\omega_x^{n-2} + g\omega_y^{n-2} - n\right] = \left[m - d\omega_x^{n-1} - f\omega_y^{n-1}\right]Z \tag{19}$$

It yields an expression of Z.

$$Z = \frac{e\omega_x^{n-2} + g\omega_y^{n-2} - n}{m - d\omega_x^{n-1} - f\omega_y^{n-1}} \tag{20}$$

We can get

$$Y = \omega_y Z \tag{21}$$

$$Y = \omega_y \frac{e\omega_x^{n-2} + g\omega_y^{n-2} - n}{m - d\omega_x^{n-1} - f\omega_y^{n-1}} \tag{22}$$

$$X = \omega_x Z \tag{23}$$

$$X = \omega_x \frac{e\omega_x^{n-2} + g\omega_y^{n-2} - n}{m - d\omega_x^{n-1} - f\omega_y^{n-1}} \tag{24}$$

□

Proving FLT , would be equivalent to proving that above expressions for X,Y,Z cannot have integer solutions.[will complete later].